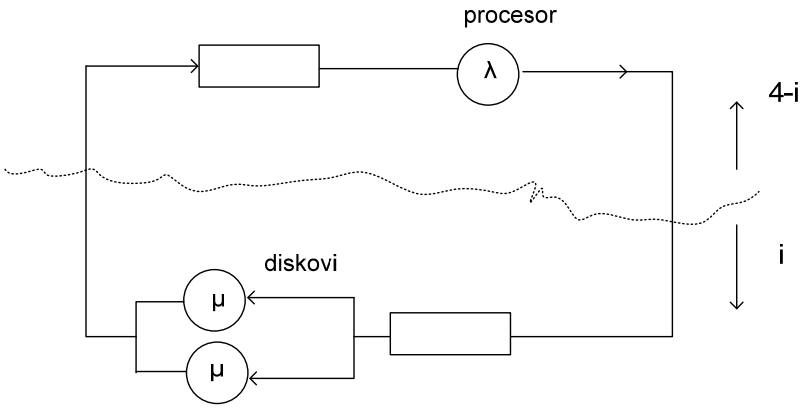
Performanse računarskih sistema
-rešenja zadataka-

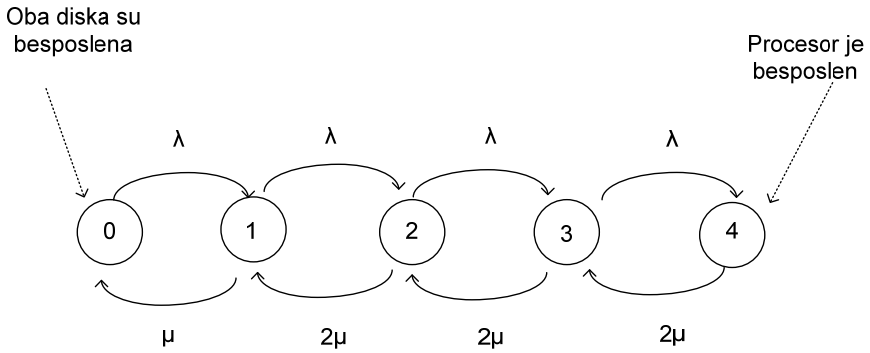
1. Pogledati predavanja.

2. a) Pogledati predavanja. Rezultat: $\frac{2}{N^2}\int_0^N (N-z)\cdot Tam(z)\cdot dz$

b) Razlikovao bi se rezultat jer bi drugi procesi svojim pristupima drugim delovima diska pomerili glavu diska na neki drugi kraj.

3. Sistem i dijagram stanja prikazani su na narednim slikama:





$\lambda = \frac{1}{s_p}, \quad \mu = \frac{1}{s_d}$

Ako količnik $\frac{\lambda}{2\mu}$ obeležimo sa ρ, tada su balansne jednačine za ovaj sistem:

$p_0 \cdot \lambda = p_1 \cdot \mu \Rightarrow p_1 = 2\rho \cdot p_0$

$p_1 \cdot \lambda = p_2 \cdot 2\mu \Rightarrow p_2 = 2\rho^2 \cdot p_0$

$p_2 \cdot \lambda = p_3 \cdot 2\mu \Rightarrow p_3 = 2\rho^3 \cdot p_0$

$p_3 \cdot \lambda = p_4 \cdot 2\mu \Rightarrow p_4 = 2\rho^4 \cdot p_0$

$p_0 + p_1 + p_2 + p_3 + p_4 = 1 \Rightarrow p_0 \cdot \left(1 + 2\rho + 2\rho^2 + 2\rho^3 + 2\rho^4\right) = 1$

$U_d = 1 - p_0 - \frac{1}{2} \cdot p_1 = \frac{183}{203} \Rightarrow p_0 + \frac{1}{2} \cdot p_1 = \frac{20}{203}$

Kako je verovatnoća da je na procesor ne čeka više od jednog procesa jednaka $p_2 + p_3 + p_4 = 1 - \left(p_0 + p_1\right) = \frac{171}{203}$, to je

$p_0 + p_1 = \frac{32}{203}$. Dalje je

$p_1 = 2 \cdot \left(\frac{32}{203} - \frac{20}{203}\right) = \frac{24}{203}$

$p_0 = \frac{32}{203} - \frac{24}{203} = \frac{8}{203}$

$\rho = \frac{1}{2} \cdot \frac{p_1}{p_0} = \frac{3}{2},\ s_p = \frac{s_d}{2 \cdot \rho} = 5ms$

Iskorišćenje procesora: $Up = 1 - p_4 = 1 - 2 \cdot \rho^4 \cdot p_0 = 1 - \frac{81}{203} = \frac{122}{203}$

Protok kroz procesor: $Xp = \frac{U_p}{s_p} \approx 120.2\, posl / \sec$

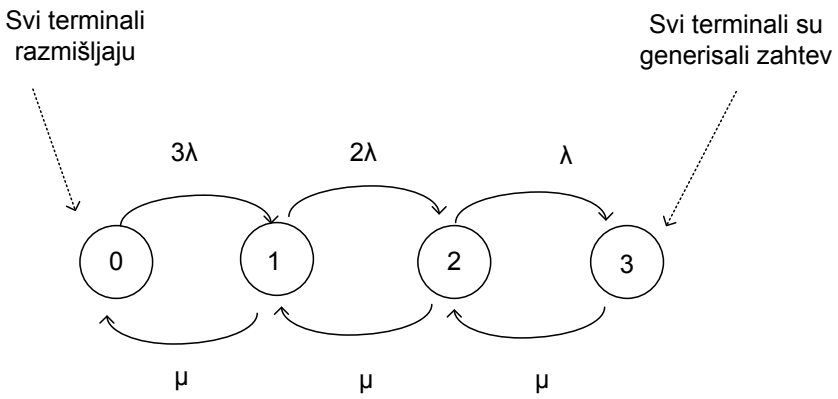
Srednji broj poslova u procesoru: $J = 4 \cdot p_0 + 3 \cdot p_1 + 2 \cdot p_2 + p_3 = 2p_0 \cdot \left(2 + 3\rho + 2\rho^2 + \rho^3\right) = \frac{230}{203} \approx 1.133$

Vreme odziva procesora: $R_p = \frac{J_p}{X_p} = 9.426ms$

Srednje vreme čekanja: $Tq = R_p - s_p = 4.426ms$

4. a) Dijagram stanja datog sistema se može prikazati na sledeći način:

Stanje ***i*** znači da je ***i*** terminala aktivno, a ostali razmišljaju i mogu generisati zahteve.



Balansne jednačine za ovaj sistem:

$$p_0 \cdot 3\lambda = p_1 \cdot \mu \Rightarrow p_1 = 3\frac{\lambda}{\mu} \cdot p_0, \ \frac{\lambda}{\mu} = \frac{\frac{1}{\overline{\theta}}}{\frac{1}{\overline{s_p}}} = \frac{s_p}{\theta} = \frac{1}{4} = \rho$$

$$p_1 \cdot 2\lambda = p_2 \cdot \mu \Rightarrow p_2 = 2\frac{\lambda}{\mu} \cdot p_1 = 6 \cdot \rho^2 \cdot p_0$$

$$p_2 \cdot \lambda = p_3 \cdot \mu \Rightarrow p_3 = \frac{\lambda}{\mu} \cdot p_2 = 6 \cdot \rho^3 \cdot p_0$$

$$p_0 + p_1 + p_2 + p_3 = 1 \Rightarrow p_0 \cdot \left(1 + 3 \cdot \rho + 6 \cdot \rho^2 + 6 \cdot \rho^3\right) = 1$$

$$p_0 = \frac{1}{1 + 3 \cdot \rho + 6 \cdot \rho^2 + 6 \cdot \rho^3} = \frac{1}{1 + \frac{3}{4} + \frac{3}{8} + \frac{3}{32}} = \frac{32}{71}$$

$$p_1 = \frac{24}{71}, \ p_2 = \frac{12}{71}, \ p_3 = \frac{3}{71}$$

Iskorišćenje procesora:

$$U = 1 - p_0 = \frac{39}{71} \approx 54.93\%$$

Vreme odziva se određuje na sledeći način : $r + \overline{\theta} = \frac{n \cdot \overline{s_p}}{U}$

Kako je $\overline{s_p} = \frac{\overline{\theta}}{4}$, to je:

$$r + \overline{\theta} = \frac{n \cdot \frac{\overline{\theta}}{4}}{U} = \frac{3}{4} \cdot \frac{\overline{\theta}}{U} \Rightarrow \overline{\theta}\left(\frac{3}{4U} - 1\right) = r$$

$$\overline{\theta} = \frac{4r \cdot U}{3 - 4U} = 328.42ms$$

$$\overline{s_p} = \frac{\overline{\theta}}{4} = 82.105ms$$

Protok kroz procesor:

$$X = \frac{U}{Sp} = 6.69\, posl/\sec$$

Prosečan broj poslova u procesorskom sistemu se može odrediti iz Little-ove formule:

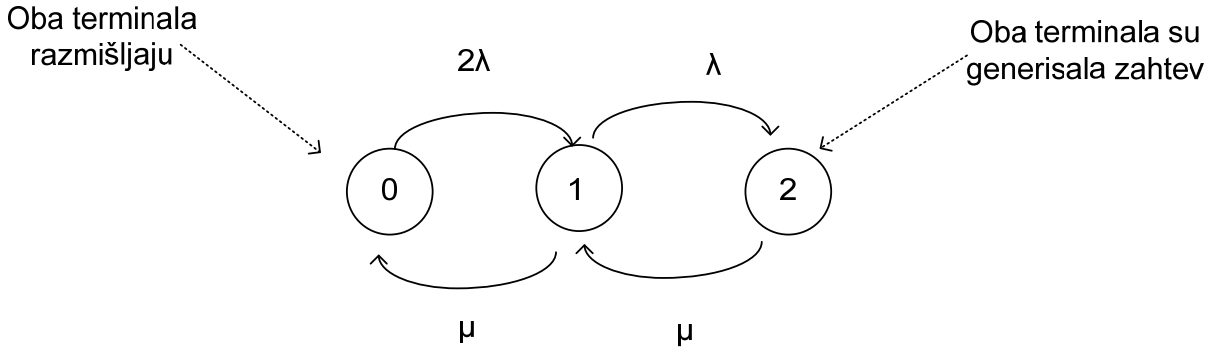
$$r = \frac{J}{X} \Rightarrow J = r \cdot X \approx 0.803$$

Kao proveru, prosečan broj poslova možemo odrediti i na drugi način (po definiciji):

$$J = p_1 + 2 \cdot p_2 + 3 \cdot p_3 = \frac{24}{71} + 2 \cdot \frac{12}{71} + 3 \cdot \frac{3}{71} = \frac{57}{71} \approx 0.803$$

b) Za sortitanje 3 puta dužeg vektora potrebno je, po datom algoritmu, 9 puta više procesorskog vremena. Pritom se sortiranje vrši jedino kada je procesor slobodan što se tiče interaktivnih zahteva.

Kada su u sistemu dva terminala, dijagram stanja izgleda kao na sledećoj slici:



Iz balansnih jednačina se određuje iskorišćenje procesora kada su dva terminal uključena:

$$p_0 \cdot 2\lambda = p_1 \cdot \mu \Rightarrow p_1 = 2\frac{\lambda}{\mu} \cdot p_0 = 2\rho \cdot p_0, \quad \frac{\lambda}{\mu} = \frac{\frac{1}{\theta}}{\frac{1}{s_p}} = \frac{s_p}{\theta} = \frac{1}{4} = \rho$$

$$p_1 \cdot \lambda = p_2 \cdot \mu \Rightarrow p_2 = \frac{\lambda}{\mu} \cdot p_1 = 2 \cdot \rho^2 \cdot p_0$$

$$p_0 + p_1 + p_2 = 1 \Rightarrow p_0 \cdot \left(1 + 2 \cdot \rho + 2 \cdot \rho^2\right) = 1$$

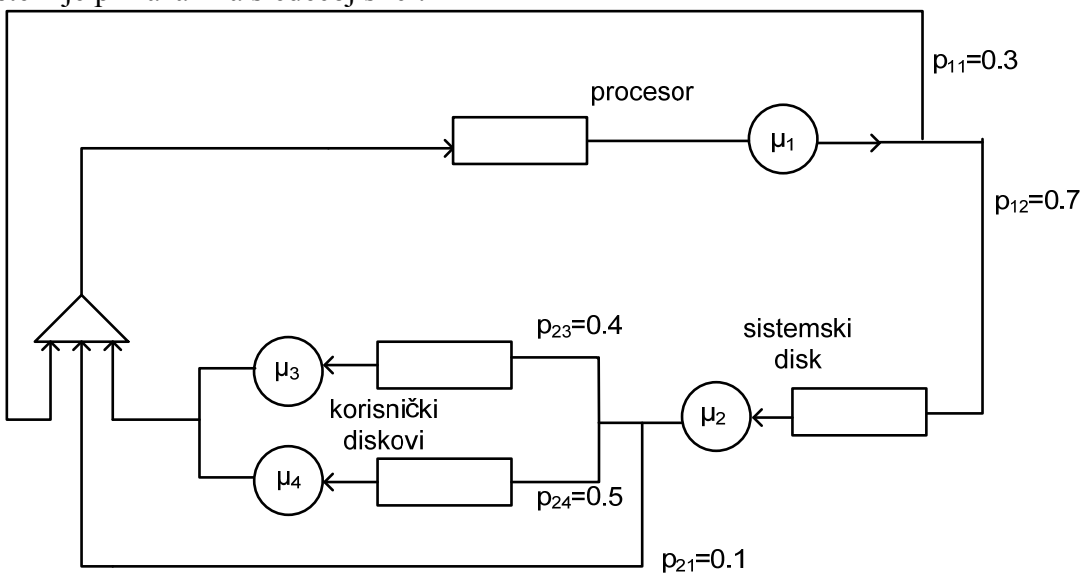
$$p_0 = \frac{1}{1 + 2 \cdot \rho + 2 \cdot \rho^2} = \frac{1}{1 + \frac{1}{2} + \frac{1}{8}} = \frac{8}{13}$$

$$U(2) = 1 - p_0 = \frac{5}{13} \approx 38.46\%$$

$$T_1 \cdot \left(1 - U(2)\right) = 9 \cdot T \cdot (1 - U(3))$$

$$T_1 = \frac{9 \cdot T(1 - U(3))}{1 - U(2)} = 180s \cdot \frac{p_0(3)}{p_0(2)} = 180s \cdot \frac{\frac{32}{71}}{\frac{8}{13}} = 131.83s$$

5. Dati sistem je prikazan na sledećoj slici:



Gordon-Newell-ove jednačine za data 4 resursa:

$$-\left(1-p_{11}\right)\mu_1 x_1 + p_{21}\mu_2 x_2 + p_{31}\mu_3 x_3 + p_{41}\mu_4 x_4 = 0$$
$$p_{12}\mu_1 x_1 - \left(1-p_{22}\right)\mu_2 x_2 + p_{32}\mu_3 x_3 + p_{42}\mu_4 x_4 = 0$$
$$p_{13}\mu_1 x_1 + p_{23}\mu_2 x_2 - \left(1-p_{33}\right)\mu_3 x_3 + p_{43}\mu_4 x_4 = 0$$
$$p_{14}\mu_1 x_1 + p_{24}\mu_2 x_2 + p_{34}\mu_3 x_3 - \left(1-p_{44}\right)\mu_4 x_4 = 0$$

$$p_{11}=0.3,\ \ p_{12}=0.7,\ \ p_{21}=0.1,\ \ p_{23}=0.4,\ \ p_{24}=0.5,\ \ p_{31}=1,\ p_{41}=1$$

ostale verovatnoće su jednake nuli.

$$\mu_1 = \frac{1}{S_p} = 200\,\mathrm{sec}^{-1},\ \mu_2 = \frac{1}{S_{sd}} = 100\,\mathrm{sec}^{-1},$$
$$\mu_3 = \frac{1}{S_{d1}} = 50\,\mathrm{sec}^{-1},\ \mu_4 = \frac{1}{S_{d2}} = 40\,\mathrm{sec}^{-1},$$

Uzimajući da je x1=1, rešavanjem sistema dobijaju se ostale vrednosti:

x_2=1.4, x_3=1.12, x4=1.75

Broj poseta pojedinim resursima:

$V1 = 1,\ \ V2 = 0.7$

$V3 = V2 \cdot 0.4 = 0.28$

$V4 = V2 \cdot 0.5 = 0.35$

Vremena servisiranja resursa:

$S_1 = S_p = 5ms,\ S_2 = S_{sd} = 10ms$

$S_3 = S_{d1} = 20ms,\ S_4 = S_{d2} = 25ms$

Podražnje za resursima:

$D_1 = V_1 \cdot S_1 = 5ms,\ D_2 = V_2 \cdot S_2 = 7ms$

$D_3 = V_3 \cdot S_3 = 5.6ms,\ D_4 = V_4 \cdot S_4 = 8.75ms$

a)

```
public class Sep08 {
  private static final int k = 4;
  public static void mva(int n) {
      double Qprev[] = new double[k];
      double R[] = new double[k];
      double Ru = 0;
      double D[] = { 5, 7, 5.6, 8.75 };
      double V[] = { 1, 0.7, 0.28, 0.35 };
      double U[] = new double[k];
      double Xj[] = new double[k];
      double X;
      for (int i = 0; i < k; i++) Qprev[i] = 0;
      for (int i = 1; i <= n; i++) {
            Ru = 0;
            for (int j = 0; j < k; j++) {
                  R[j] = D[j] * (1 + Qprev[j]);
                  Ru = Ru + R[j];
            }
            X = i / (Ru);
            for (int j = 0; j < k; j++) {
                  U[j] = D[j] * X;
                  Qprev[j] = X * R[j];
                  Xj[j] = X * V[j];
            }
      }
      System.out.println("Iskoriscenje procesora: " + U[0]);
      System.out.println("Iskoriscenje sistemskog diska: " + U[1]);
      System.out.println("Iskoriscenje 1. korisnickog diska: " + U[2]);
      System.out.println("Iskoriscenje 2. korisnickog diska: " + U[3]);
      System.out.println("Protok kroz procesor: " + 1000 * Xj[0]);
      System.out.println("Protok kroz sistemski disk: " + 1000 * Xj[1]);
      System.out.println("Protok kroz 1. korisnicki disk: " + 1000 * Xj[2]);
      System.out.println("Protok kroz 2. korisnicki disk: " + 1000 * Xj[3]);
      System.out.println("Vreme odziva= " + Ru + "ms");
      }
      public static void main(String[] args) {
            int n = Integer.parseInt(args[0]);
            mva(n);
      }
}
```

Izlaz programa za argument N=4:

```
Iskoriscenje procesora: 0.42144065141886644
Iskoriscenje sistemskog diska: 0.590016911986413
Iskoriscenje 1. korisnickog diska: 0.47201352958913034
Iskoriscenje 2. korisnickog diska: 0.7375211399830163
Protok kroz procesor: 84.28813028377328
Protok kroz sistemski disk: 59.001691198641296
Protok kroz 1. korisnicki disk: 23.60067647945652
Protok kroz 2. korisnicki disk: 29.500845599320648
Vreme odziva= 47.45626681400073ms
```

b)

#	X1=1	X2=1.4	X3=1.12	X4=1.75	
0	1	1	1	**1**	**= G(0)**
1	1	2.4	3.52	**5.27**	**= G(1)**
2	1	4.36	8.3024	**17.5249**	**= G(2)**
3	1	7.104	16.402688	**47.071263**	**= G(3)**
4	1	10.9456	29.31661056	**111.69132081**	**= G(4)**

Iskorišćenje procesora: $U_p = g \cdot x_1 = 0.4214406514188665$

Iskorišćenje sistemskog diska: $U_{sd} = g \cdot x_2 = 0.590016911986413$

Iskorišćenje diska 1: $U_{d1} = g \cdot x_3 = 0.4720135295891305$

Iskorišćenje diska 2: $U_{d2} = g \cdot x_4 = 0.7375211399830164$

Protok kroz procesor: $X_p = \frac{U_p}{s_p} = 84.2881302837733$

Protok sistemski disk: $X_{sd} = \frac{U_{sd}}{s_{sd}} = 59.0016911986413$

Protok kroz disk 1: $X_{d1} = \frac{U_{d1}}{s_{d1}} = 23.600676479456524$

Protok kroz disk 2: $X_{d2} = \frac{U_{d2}}{s_{d2}} = 29.500845599320655$

Vreme odziva: $R = \frac{n}{X} = \frac{4}{X_p} \approx 45ms$

c) Verovatnoća da je red za čekanje neprazan znači da u sistemu drugog korisničkog diska postoji bar dva zahteva (jedan koji se obrađuje i bar jedan koji čeka). Stoga tražena verovatnoća iznosi:

$P = x_3^2 \cdot \frac{G(2)}{G(4)} = 0.1968$