Računarska grafika 2

Pregled vektorske i matrične algebre

Osnove o vektorima

$$V = \langle V_1, V_2, \ldots, V_n \rangle$$

$$aV = Va = \langle aV_1, aV_2, \ldots, aV_n \rangle$$

$$P + Q = \langle P_1 + Q_1, P_2 + Q_2, \ldots, P_n + Q_n \rangle$$

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \ldots \\ V_n \end{bmatrix}$$

$$\|V\| = \sqrt{\sum_{i=1}^{n} V_i^2}$$

$$V^T = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix}$$

Osnove o vektorima

Skalarni proizvod dva vektora

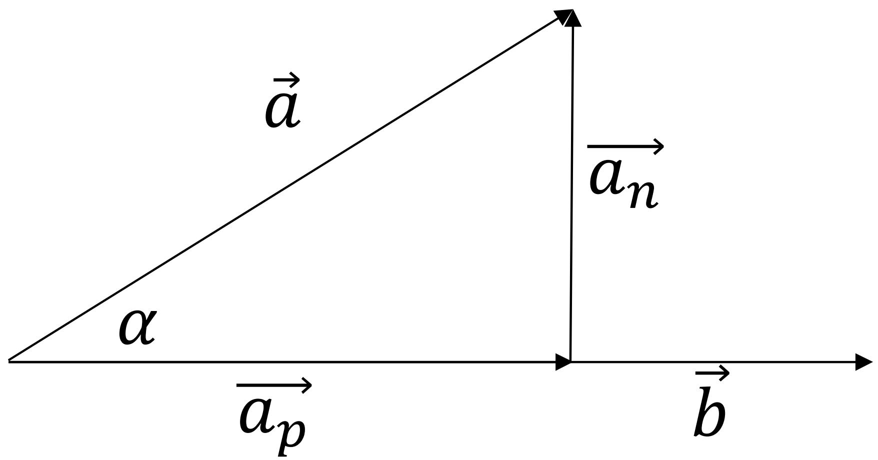
$$P \cdot Q = \sum_{i=1}^{n} P_i \cdot Q_i$$

$$P \cdot Q = \|P\| \, \|Q\| \cos \alpha$$

$$P^T Q = \begin{bmatrix} P_1 & P_2 & \cdots & P_n \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ \cdots \\ Q_n \end{bmatrix}$$

Osnove o vektorima

Projekcija i normalna komponenta



$\overrightarrow{a_p}$ - projekcija vektora $\vec{a}$ na vector $\vec{b}$

$\overrightarrow{a_n}$ - normalna komponenta vektora $\vec{a}$

Osnove o vektorima

$\overrightarrow{a_p} = A \ * \overrightarrow{i_b}$, gde je $\overrightarrow{i_b}$ jedinični vektor u pravcu vekora $\vec{b}$

$$A = \|\vec{a}\| * \cos\alpha = \vec{a} \cdot \overrightarrow{i_b}$$

$$\overrightarrow{a_p} = \left(\vec{a} \cdot \overrightarrow{i_b}\right) * \overrightarrow{i_b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|} * \frac{\vec{b}}{\|\vec{b}\|} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|^2} * \vec{b}$$

$$\overrightarrow{a_n} = \vec{a} - \overrightarrow{a_p} = \vec{a} - \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|^2} * \vec{b}$$

Osnove o vektorima

Vektorski proizvod dva 3D vektora

$$P \times Q = \langle P_y Q_z - P_z Q_y, P_z Q_x - P_x Q_z, P_x Q_y - P_y Q_x \rangle$$

$$P \times Q = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} \qquad \begin{matrix} \mathrm{i} = \langle 1,0,0 \rangle \\ \mathrm{j} = \langle 0,1,0 \rangle \\ \mathrm{k} = \langle 0,0,1 \rangle \end{matrix}$$

$$P \times Q = \begin{bmatrix} 0 & -P_z & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{bmatrix} \begin{bmatrix} Q_x \\ Q_y \\ Q_z \end{bmatrix}$$

Osnove o matricama

$$\mathrm{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \quad \mathrm{M}^{\mathrm{T}} = \begin{bmatrix} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{bmatrix}$$

$$a\mathrm{M} = \mathrm{M}a = \begin{bmatrix} aM_{11} & aM_{12} & aM_{13} \\ aM_{21} & aM_{22} & aM_{23} \\ aM_{31} & aM_{32} & aM_{33} \end{bmatrix} \qquad \mathrm{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{M} + \mathrm{N} = \begin{bmatrix} M_{11} + N_{11} & M_{12} + N_{12} & M_{13} + N_{13} \\ M_{21} + N_{21} & M_{22} + N_{22} & M_{23} + N_{23} \\ M_{31} + N_{31} & M_{32} + N_{32} & M_{33} + N_{33} \end{bmatrix}$$

Osnove o matricama

Teoreme (bez dokazivanja).
M je matrica m × n, N je matrica n × p

$\mathrm{M} + \mathrm{N} = \mathrm{N} + \mathrm{M}$
$(\mathrm{M} + \mathrm{N}) + \mathrm{P} = \mathrm{N} + (\mathrm{M} + \mathrm{P})$
$a(b\mathrm{M}) = (ab)\mathrm{M}$
$a(\mathrm{M} + \mathrm{N}) = a\mathrm{M} + a\mathrm{N}$
$(a + b)\mathrm{M} = a\mathrm{M} + b\mathrm{M}$
$(\mathrm{MN})^{\mathrm{T}} = \mathrm{N}^{\mathrm{T}}\mathrm{M}^{\mathrm{T}}$