Računarska grafika 2

13M111RG2

1 Pregled vektorske i matrične algebre

Osnove o vektorima (podsetnik)

$$\mathbf{V} = \langle V_1, V_2, \ldots, V_n \rangle \qquad a\mathbf{V} = \mathbf{V}a = \langle aV_1, aV_2, \ldots, aV_n \rangle$$

$$\mathbf{P} + \mathbf{Q} = \langle P_1 + Q_1, P_2 + Q_2, \ldots, P_n + Q_n \rangle$$

$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \ldots \\ V_n \end{bmatrix}$$

$$\|\mathbf{V}\| = \sqrt{\sum_{i=1}^{n} V_i^{\,2}}$$

$$\mathbf{V}^{\mathbf{T}} = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix}$$

Osnove o vektorima (podsetnik)

Skalarni proizvod dva vektora

$$\mathbf{P} \cdot \mathbf{Q} = \sum_{i=1}^{n} P_i \cdot Q_i$$

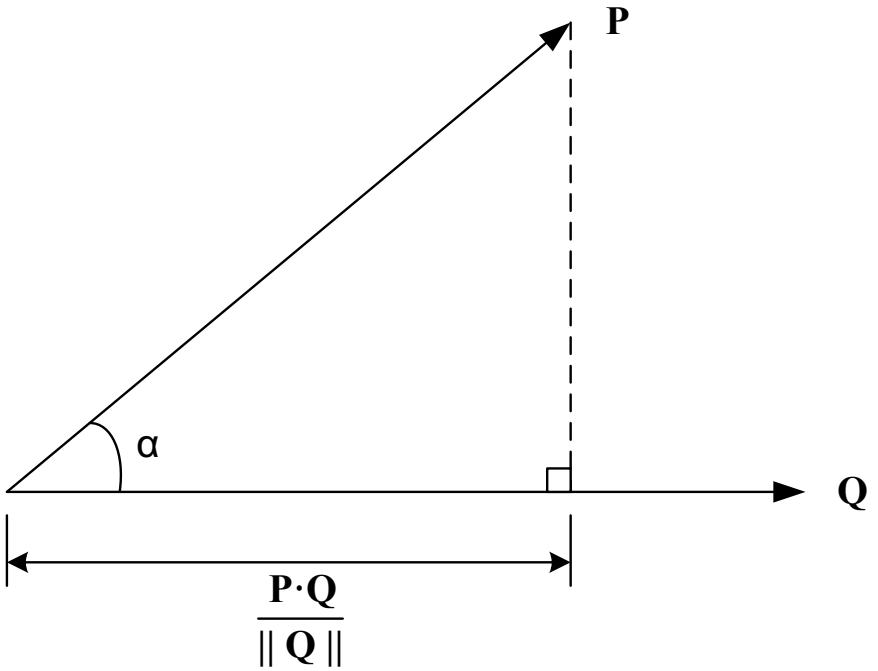
$$\mathbf{P}^{\mathbf{T}}\mathbf{Q} = \begin{bmatrix} P_1 & P_2 & \dots & P_n \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ \dots \\ Q_n \end{bmatrix}$$

$$\mathbf{P} \cdot \mathbf{Q} = \left\| \mathbf{P} \right\| \left\| \mathbf{Q} \right\| \cos \alpha$$

Osnove o vektorima (podsetnik)

Projekcija i normalna komponenta

$$proj_Q \mathbf{P} = \frac{\mathbf{P} \cdot \mathbf{Q}}{\|\mathbf{Q}\|^2} \mathbf{Q}$$



$$perp_Q \mathbf{P} = \mathbf{P} - \frac{\mathbf{P} \cdot \mathbf{Q}}{\|\mathbf{Q}\|^2} \mathbf{Q}$$

Osnove o vektorima (podsetnik)

Vektorski proizvod dva 3D vektora

$$\mathbf{P}\times\mathbf{Q}=\left\langle P_yQ_z-P_zQ_y,P_zQ_x-P_xQ_z,P_xQ_y-P_yQ_x\right\rangle$$

$$\mathbf{P}\times\mathbf{Q}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\P_x&P_y&P_z\\Q_x&Q_y&Q_z\end{vmatrix}\qquad\begin{aligned}\mathbf{i}&=\langle 1,0,0\rangle\\\mathbf{j}&=\langle 0,1,0\rangle\\\mathbf{k}&=\langle 0,0,1\rangle\end{aligned}$$

$$\mathbf{P}\times\mathbf{Q}=\begin{bmatrix}0&-P_z&P_y\\P_z&0&-P_x\\-P_y&P_x&0\end{bmatrix}\begin{bmatrix}Q_x\\Q_y\\Q_z\end{bmatrix}$$

Osnove o matricama (podsetnik)

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \qquad \mathbf{M}^{\mathbf{T}} = \begin{bmatrix} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{bmatrix} \qquad \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a\mathbf{M} = \mathbf{M}a = \begin{bmatrix} aM_{11} & aM_{12} & aM_{13} \\ aM_{21} & aM_{22} & aM_{23} \\ aM_{31} & aM_{32} & aM_{33} \end{bmatrix}$$

$$\mathbf{M} + \mathbf{N} = \begin{bmatrix} M_{11} + N_{11} & M_{12} + N_{12} & M_{13} + N_{13} \\ M_{21} + N_{21} & M_{22} + N_{22} & M_{23} + N_{23} \\ M_{31} + N_{31} & M_{32} + N_{32} & M_{33} + N_{33} \end{bmatrix}$$

Osnove o matricama (podsetnik)

Teoreme (bez dokazivanja).
M je matrica m × n, N je matrica n × p

$$\mathrm{M}+\mathrm{N}=\mathrm{N}+\mathrm{M}$$

$$(\mathrm{M}+\mathrm{N})+\mathrm{P}=\mathrm{N}+(\mathrm{M}+\mathrm{P})$$

$$a(b\mathrm{M})=(ab)\mathrm{M}$$

$$a(\mathrm{M}+\mathrm{N})=a\mathrm{M}+a\mathrm{N}$$

$$(a+b)\mathrm{M}=a\mathrm{M}+b\mathrm{M}$$

$$(\mathrm{MN})^{\mathrm{T}}=\mathrm{N}^{\mathrm{T}}\mathrm{M}^{\mathrm{T}}$$

Za ortonormalne matrice važi:

$$\mathrm{M}^{\mathrm{T}}=\mathrm{M}^{-1}$$